

Here is a list of important math rules you should know. We may not use all of them directly, or I may omit some to save time, but they are generally helpful in econometrics work.

Summation Rules

Let x and y be vectors of length n .

1. Summation definition: $\sum_{i=1}^n x_i \equiv x_1 + x_2 + \cdots + x_n$
2. The sum of $x + y$ is the same as the sum of x plus the sum of y : $\sum_i (x_i + y_i) = \sum_{i=1} x_i + \sum_i y_i$
3. For any constant c , the sum of $c \times x$ is the same as c times the sum of x : $\sum_i cx_i = c \sum_i x_i$
4. In general, the sum of x times y is not equal to the sum of x times the sum of y :

$$\sum_i x_i y_i \neq \sum_i x_i \sum_i y_i$$

Variance Rules

- The variance of a constant is zero: $Var(c) = 0$
- The variance of a constant times a random variable: $Var(cX) = c^2 Var(X)$
- The variance of a constant plus a random variable: $Var(c + X) = Var(X)$
- The variance of the sum of two random variables: $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$

Covariance Rules

- The covariance of a random variable with a constant is 0: $Cov(X, c) = 0$
- The covariance of a random variable with itself is its variance: $Cov(X, X) = Var(X)$
- Constants can be brought outside of the covariance: $Cov(X, cY) = cCov(X, Y)$
- If Z is a third random variable, then: $Cov(X, Y + Z) = Cov(X, Y) + Cov(X, Z)$

plim Rules

plim stands for probability limit, that is: $plim_{n \rightarrow \infty} \hat{\theta}_n = \theta$. So as the sample size n increases, the estimator $\hat{\theta}_n$ converges in probability to the true value θ .

Let c be a constant. Let x_n and y_n be sequences of random variables where $plim(x_n) = x$ and $plim(y_n) = y$

1. The probability limit of a constant is the constant: $plim(c) = c$
2. $plim(x_n + y_n) = x + y$
3. $plim(x_n y_n) = xy$
4. $plim\left(\frac{x_n}{y_n}\right) = \frac{x}{y}$
5. $plim(g(x_n, y_n)) = g(x, y)$ for any function $g()$

Expectations Rules

Let A and B be random variables, and let c be a constant.

1. $\mathbb{E}[A + B] = \mathbb{E}[A] + \mathbb{E}[B]$
2. In general, $\mathbb{E}[AB] \neq \mathbb{E}[A]\mathbb{E}[B]$
3. Constants can pass outside of an expectation: $\mathbb{E}[cA] = c\mathbb{E}[A]$
4. Since $\mathbb{E}[A]$ is a constant, then: $\mathbb{E}[B\mathbb{E}[A]] = \mathbb{E}[A]\mathbb{E}[B]$

Conditional Expectations Rules

If the conditional expectation of something is a constant, then the unconditional expectation is that same constant:

If $\mathbb{E}[A|B] = c$, then $\mathbb{E}[A] = c$.

Why? The **law of iterated expectations**:

$$\begin{aligned}\mathbb{E}[A] &= \mathbb{E}[\mathbb{E}[A|B]] \\ &= \mathbb{E}[c] \\ &= c\end{aligned}$$

Log Rules

1. $\log_e(e) = 1$
2. $\log(ab) = \log(a) + \log(b)$
3. $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$
4. $\log(a^b) = b \times \log(a)$